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Cascade of elimination and emergence of pure cooperation in coevolutionary games on networks

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We propose a coevolutionary game to study cooperative behavior in the presence of catastrophic phenomenon. We incorporate tolerance to elimination of individuals in network games where individuals update their strategies synchronously, and there are no birth of individuals and stochastic effects. We find that an avalanchelike death process can arise when defection strategies exist and individuals are vulnerable to deficiency of profits. Strikingly, we observe that, after such a cascading process terminates, cooperators are the sole survivors regardless of the game types and of the connection patterns among individuals as determined by the network topology. Cooperation thus becomes the optimal strategy and absolutely outperforms defection. Our results can yield insights into evolution of cooperation in the presence of catastrophic events in social and natural systems.

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Evolutionary games have been a powerful tool to study a variety of self-organized behaviors in natural, social, and economical systems [1–4]. For example, understanding how cooperation emerges among selfish individuals has been a challenging problem because of the social dilemma that disfavors cooperation [5–8]. Games such as the prisoner's dilemma games (PDGs) [9], the snowdrift games (SGs) [10], and the public goods games (PGGs) [11] have been used to model interactions among selfish individuals and how the social dilemma can be resolved through self-organization. A number of mechanisms have been discovered that facilitate cooperation, which include reputation and punishment [12–17], network reciprocity [18–22], success-driven migration [23], etc.

So far, tolerance of individuals to elimination or death in the paradigm of games has received relatively little attention. A representative example relevant to the tolerance is the bankruptcy of agents in economical system. For any profitable agent, a lowest amount of profit should be maintained, which comes from the interactions with other agents in a certain time period for continuous investment into the future. In ecosystems, individuals compete and cooperate for essential life-sustaining resources to be alive. If the lowest requirement of resources cannot be gained, individuals will die. In this regard, we incorporate an elimination mechanism into the gaming rules to better mimicking evolution of cooperative behavior in real systems. In particular, we assign a tolerance parameter to every individual in the network, which is the lowest payoff needed for an indiviudal to survive. Considering diversity in social and biological systems, individuals may have their own tolerance, determined by their own features. The number of interactions of an individual is a prominent feature to distinguish it from others so that can be appropriately used to define the tolerance. In the game on networks, the death of an individual leads to the remove of its nodes together with the loss of all its connections with others. As a result, the games and network coevolve induced by elimination with respect to survival tolerance. Not that our model is different from existent coevolutionary games in which players can adaptively choose their counterparts to maximize their benefits [24–26]. While our coevolutionary game considers the low-benefit-induced death and changes in interacting networks, where the death of agents is much faster than the generation of new ones in a certain time period so that the birth process can be neglected.

Our main finding is that in the presence of defectors, a cascading process of death of individuals can occur in relatively short time, which can even spread to the whole network, leading to complete extinction. Strikingly, we find that a complete cooperation state emerges after a cascade terminates and the exclusive survivors are cooperators, which holds regardless of the type of games and network topology. This finding strongly suggests that defectors, despite their temporary advantages, are extremely vulnerable to the occurrence of catastrophic behavior. Cooperation becomes the optimal strategy to maximize benefit and avoid death for an individual, naturally resolving the social dilemma of profit versus cooperation. Our results can yield insight into the catastrophic events in economical and ecosystems. For example, during the recent economic recession, the ceaseless bankruptcy of profit organizations and institutes is a typical cascading process where high-risk investments as defection behavior decrease the capacity of agents to resist deficiency and trigger the outbreak of bankruptcy cascades. Our model may also related with species extinctions in a relatively short period.

We consider three typical games: PDG, SG, and PGG. The main ingredients of these games are as follows. (i) In a PDG, both players are offered a reward R for mutual cooperation and a punishment P for mutual defection. If one cooperates but the other defects, the defector (D) gets the highest payoff T, while the cooperator (C) gets the lowest payoff S. The payoff rank for PDG is thus $T>R>P>S$. (ii) In a SG, the payoff rank is $T>R>S>P$. (iii) PGG is played by a group of players. The total reward by the sole contribution of C's is enhanced by a factor η and equally distributed in the group.

At each time step, the actual payoff gained by any individual is the summation of payoffs from all interactions. At each iteration, there are three processes.

(i) Game playing and payoffs. For PDG, we follow previous studies and use the rescaled parameters $R=1$, $T=b(b>1)$, and $S=P=0$ [18,27]. For SG, we set $R=1$, $T=1+r$, $S=1-r$, and $P=0$, with $0<r<1$ [20]. For PGG, in

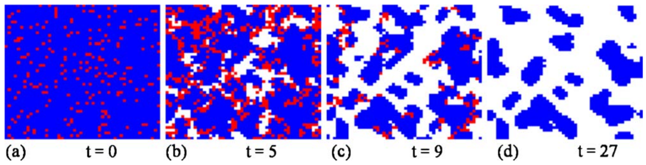

FIG. 1. (Color online) For PDG on a two-dimension lattice with four neighbors (2D4n), evolution of spatial patterns for $b=1.1$ at (a) $t=0$, some D's are randomly placed in the ocean of C's; (b) $t=5$, there is an increase in the number of D's and both C's and D's begin to die; (c) $t=9$, large numbers of D's die and clusters of C's begin to form; (d) $t=27$, D's become extinct and the lattice is shared by C clusters and empty sites. After the extinction of D's, the patterns become time invariant. Similar results have been obtained for SG and PGG. The lattice size is 50×50 and all sites are occupied initially. The color coding is red (light gray) for D's, blue (dark gray) for cooperators, and white for empty sites. At $t=0$, 10% of the occupants are D's (randomly distributed). For $t>27$, the spatial pattern is invariant.

an arbitrary group formed by node x and its neighbors, the payoffs of a D and a C are $P(D)=c\eta n(C)/(k_x+1)$ and $P(C)=P(D)-c$, respectively, where η is the enhancement parameter, $n(C)$ is the number of C's in the group and k_x is the number of neighbors of node x. c is set to be unity [22].

(ii) Failure and individual removal. At each iteration, the node that hosts individual i and all its links will be removed if $P_i<T_i$, where T_i is the tolerance to death, defined as

$$T_i = \begin{cases} \alpha P_i^N = \alpha k_i, & \text{for PDG and SG} \\ \alpha P_i^N = \alpha(\eta-1)(k_i+1), & \text{for PGG,} \end{cases} \quad (1)$$

where P_i^N is the normal payoff when the system is in a healthy state in which all individuals are C's, $0\leq\alpha\leq 1$ is a tolerance parameter, and k_i is the number of neighbors of i. For $\alpha=1$ and $\alpha=0$, individuals have zero tolerance and complete tolerance to breakdown, respectively.

(iii) Strategy updating. At each time step, i randomly chooses a survived neighbor j and imitates js strategy with the probability [28] $W_{i\to j}=1/(1+\exp[-(P_j-P_i)/\mathcal{K}])$, where $\mathcal{K}=0.1$ represents the uncertainties in assessing the payoffs.

In addition, all strategies are synchronously updated and external noise is absent. Since the failure and strategy updating processes occurs in parallel, the ratio of their occurrence rates is fixed, as in Ref. [29].

We first examine the evolution of spatial patterns, as shown in Figs. 1(a)–1(d). The general observation is that, when defecting strategies are practiced, the occurrence of large-scale cascading processes is common and, in order to survive, an individual needs to cooperate persistently. The size s_d of dead individuals, defined as the number of removed nodes normalized by the network size, depends on the tolerance parameter α, the temptation to defect (b or r), and the enhancement factor η.

In general, the spreading of defection strategy and the loss of interactions induce the cascade of death. The higher payoff of D's can induce the imitation of neighboring C's or the death of neighboring C's due to insufficient cooperation, which establishes a negative feedback mechanism, leading to the reduction in D's payoffs and their eventually death. Once C's turn to be D's, the defection strategy spreads and further death can follow until the emergence of C clusters, at the boundary of which, C's receive sufficient mutual cooperation to resist both invasion of D's and insufficient payoffs to death.

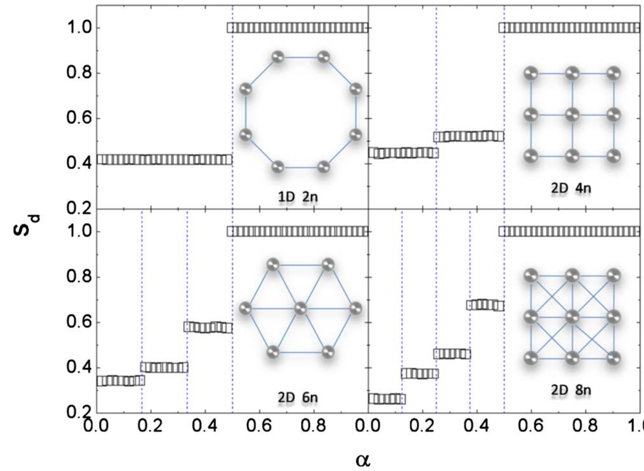

FIG. 2. (Color online) For PDG on four types of regular networks (1D2n, 2D4n, 2D6n, and 2D8n), the fraction of failed (dead) individuals s_d as a function of the tolerance parameter α. The dashed vertical lines are theoretical predictions for various transitions between distinct states, including the extinction transition. The network size is 100×100 and all data points are obtained after s_d becomes a constant. For the 1D2n lattice, $\alpha_c=1/2$, which separates two steps in s_d. For the 2D4n lattice, $\alpha_c=1/4, 1/2$, corresponding to three steps. Similarly, for the 2D6n lattice, we have $\alpha_c=1/6, 1/3, 1/2$, which separate four steps. For the 2D8n lattice, we have $\alpha_c=1/8, 1/4, 3/8, 1/2$, so there are five steps. Since all survivors are C's, their number N_c as a function of α displays step structures as well because of the relation $N_c=N(1-s_d)$.

To obtain a quantitative understanding of the cascading dynamics, we investigate the dependence of s_d on the tolerance parameter α for three games on four types of lattices. As shown in Fig. 2, for PDG, we observe step structures for all lattices but different numbers of steps for different lattices. A striking phenomenon is that the transition from a survival state to an extinction state occurs at the critical value $\alpha_c=0.5$, regardless of the lattice type, temptation to defection and the initial fraction of D's. Similar results have been obtained for SG with the same value of α_c. For PGG, because of the intrinsic group interactions, the behavior of s_d versus α is somewhat different from those with PDG and SG. However, the phenomenon of transition to extinction persists, as shown in Fig. 3. We observe that, for PGG, except for the 1D2n lattice, there are no clear step structures and the transition points differ for different lattices.

The structures of the "minimal" surviving clusters can be used to explain the transition to extinction, as shown schematically in Fig. 3. Their stabilities can be assessed by calculating the payoffs of individuals in the respective clusters. For example, for the 1D2n lattice, the two C's payoff is $P_i=1$ and their tolerance is $T_i=\alpha k_i=2\alpha$. For $\alpha<0.5$, we

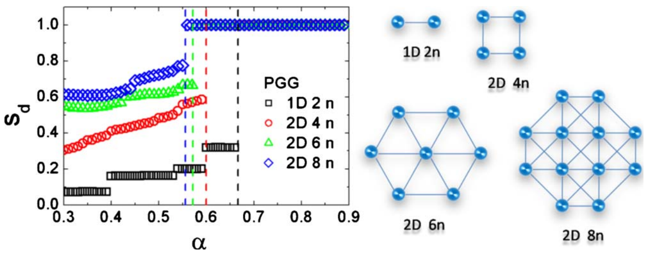

FIG. 3. (Color online) For PGG, dependence of s_d on tolerance parameter α for the four types of networks as in Fig. 2. The dashed vertical lines are theoretical predictions for the extinction-transition points: $\alpha_c^{1D2n}=2/3$, $\alpha_c^{2D4n}=3/5$, $\alpha_c^{2D6n}=4/7$, and $\alpha_c^{2D8n}=5/9$. In the right panel, four smallest surviving clusters in four types of lattices for α slightly below the critical value α_c are shown for all three games. These clusters determine the transition point to extinction. The nodes inside the cluster are more stable than those at boundaries. For $\alpha > \alpha_c$, nodes at boundaries die out and the clusters disappear.

have $P_i > T_i$ so that both C's will survive and the cluster is stable. Similarly, all clusters in Fig. 3 are stable for $\alpha < 0.5$. For $\alpha > 0.5$, there are no longer survivable structures.

The transition associated with PGG can be understood similarly. For an arbitrary surviving node i, its payoff satisfies $P_i > T_i$. Combining the condition $P_i < T_i$ and Eq. (1), we have $P_i=(\eta-1)(k_i'+1) > \alpha(\eta-1)(k_i+1)=T_i$, where k_i is the original degree of i and k_i' is the remaining degree in the aftermath of the cascading event. The transition point is then given by $\alpha_c=(k_i'+1)/(k_i+1)$, which is independent of η. Since for PGG, the smallest stable cluster has the same structure as that for PDG, α_c is also determined by the cluster structure in Fig. 3. A common property among these minimal cluster structures is that k_i' for any node is larger than or equal to $k_L/2$, where k_L is the node degree of the original lattice. Since nodes with more remaining connections are more stable for identical original degrees, the extinction-transition point α_c is determined by the nodes at the boundary. We then have $\alpha_c=(k_L+2)/[2(k_L+1)]$. These predictions are verified in Fig. 3.

We can consequently explain the presence of step structures in Fig. 2 by examining the condition for survival: $P_i=k_i' > \alpha k_i=T_i$, or $\alpha_c=k_i'/k_i$. Because the remaining degree k_i' satisfies $k_i' \le k_i$, its possible values are $1,2,\ldots,k_i$. However, since no stable cluster exists for $\alpha > 0.5$, there is an additional constraint for k_i': $k_i' \le k_i/2$. All possible values of k_i' determine the numbers of steps in Fig. 2. These predictions agree with simulation results, as shown in Fig. 2.

We next study the dependence of s_d and surviving strategies on two key game parameters, α and $b(r)$ or η for scale-free networks [30]. Figures 4(a)–4(c) show the contour plots of s_d in the two-dimensional parameter plane for PDG, SG, and PGG, respectively. Analogous to the observation on regular lattices, there exist two exclusive asymptotic phases: extinction for large values of α and b (r, small values of η) and a survival phase in which only C's can survive. C's and D's cannot coexist for any parameter combinations. Different from regular networks, for complex networks, s_d is more sensitive to the variation in α and the s_d versus α curve tends to be continuous. This is due to the fact that complex topology provides a richer spectrum of individual tolerances due to the diversity of node degrees. The boundary between extinction and survival depends on parameters. However, when cooperation is facilitated by small values of b and r and large values of η, the boundary is solely determined by the network structure, which can then be treated by a stability analysis. For example, the death of a vast majority of nodes with smallest degree can trigger extinction. Their stability in PGG can be written as $\alpha_c=(k_{min}'+1)/(k_{min}+1)$, where k_{min}' is the remaining degree. For large η, if α is reduced such that the single interaction cannot provide enough payoff for the individual to sustain, extinction will arise. Thus, $k_{min}'=1$ and $\alpha_c=2/(k_{min}+1)=4/(2+\langle k\rangle)$, where the average degree $\langle k\rangle=2k_{min}$ for a standard scale-free network [30]. Similarly, for PDG and SG, we have $\alpha_c=2/\langle k\rangle$, which is valid in the regime of small temptation to defection and large initial fraction of D's. The analysis is well supported by numerical results, as shown in Fig. 4(d). The stable cluster possesses a star-like structure, as indicated in Fig. 4(d). We also investigate the effect of noise $\mathcal{K}$ and the initial density of defector $\rho_D(0)$ on the fraction of failed individuals s_d. Their effects are exemplified by considering PGG, as shown in Fig. 5. We see that larger $\rho_D(0)$ leads to higher values of s_d, but the critical value α_c at which transition to extinction occurs is regardless of $\rho_D(0)$ [Fig. 5(a)]. While the noise $\mathcal{K}$ shows little influence to s_d, as shown in Fig. 5(b).

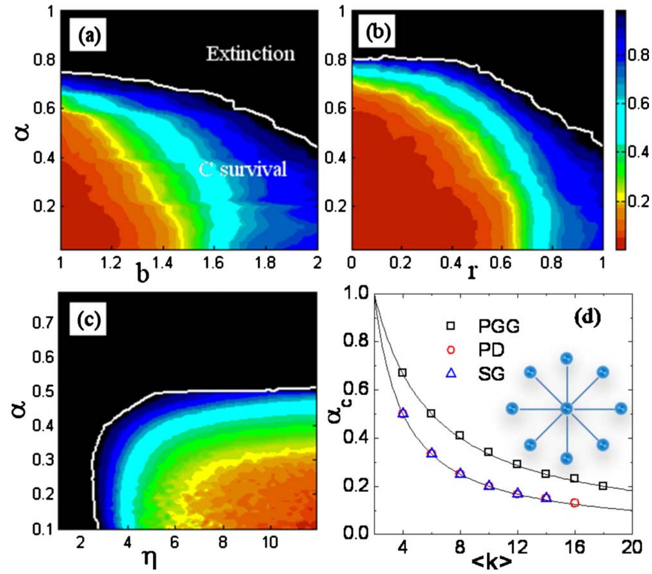

FIG. 4. (Color online) For (a) PDG, (b) SG, and (c) PGG on scale-free networks, dependence of death size s_d on tolerance parameter α and game parameters b, r, and η, respectively. The initial fraction of D's is 0.1 for PDG and SG, and 0.5 for PGG. There are two distinct asymptotic phases: extinction and survival of C's. The boundaries between the two phases are marked by the white curves. In the whole parameter space, D's cannot survive. (d) The dependence of extinction boundary α_c on the average degree $\langle k\rangle$ for the three games, where the initial fraction of D's is 0.85, $b=1.01$, and $r=1.01$ (for PDG and SG, respectively) or $\eta=10$ (for PGG). The star graph is a typical survivable C cluster when α is close to the boundary α_c. The network size is 1000. Ensemble average is based on ten network realizations and ten independent gaming processes for each network realization.

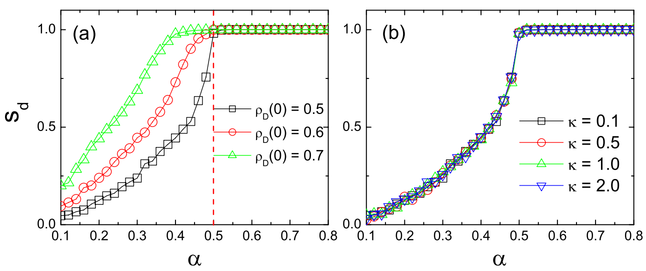

FIG. 5. (Color online) For PGG, dependence of s_d on tolerance parameter α for (a) different values of initial density $\rho_D(0)$ of defectors and (b) different values of noise $\mathcal{K}$ on the scale-free network. In (a), $\mathcal{K}$ is fixed to 0.1, and in (b), $\rho_D(0)$ is fixed to 0.1. The network size is 1000, $\langle k \rangle = 10$ and $\eta = 10$. α_c is denoted by the dashed line.

In summary, we have proposed a coevolutionary game to investigate catastrophic behavior and evolution of cooperation, and have found two generic phenomena that do not depend on details such as the network topology and game types: (1) defection strategies for temporal high payoff can result in large-scale failures or even the collapse of the entire system and (2) the optimal strategy for surviving catastrophic failures is cooperation, which are valid for a low noise level in the imitation. Defection strategies can trigger a negative feedback mechanism that weakens the viability of defectors and leads to their death ultimately. In contrast, cooperation can survive eventually in the form of clusters that resist deficit as well as the invasion of defectors. These results suggest that in order to sustain the normal functioning of the system and to maximize individual individuals' gain, cooperation is the optimal strategy. These provide insights into, for example, the phenomenon of large-scale bankruptcy witnessed during the recent global economical recession.

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